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TeV scale 5D $\mathrm{SU}(3)_W$ unification and the fixed point anomaly cancellation with chiral split multiplets

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ABSTRACT: A possibility of 5D gauge unification of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ in $\mathrm{SU}(3)_W$ is examined. The orbifold compactification allows fixed points where $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ representations can be assigned. We present a few possibilities which give long proton lifetime, top-bottom mass hierarchy from geometry, and reasonable neutrino masses. In general, these *chiral models* can lead to fixed point anomalies. We can show easily, due to the simplicity of the model, that these anomalies are cancelled by the relevant Chern-Simons terms for all the models we consider. It is also shown that the fixed point U(1)-graviton-graviton anomaly cancels without the help from the Chern-Simons term. Hence, we conjecture that the fixed point anomalies can be cancelled if the effective 4D theory is made anomaly free by locating chiral fermions at the fixed points.

KEYWORDS: Field Theories in Higher Dimensions, Discrete and Finite Symmetries, Anomalies in Field and String Theories, GUT.

Contents

1. Introduction

It is believed that the supersymmetric extension of the standard model predicts a unification of gauge coupling constants with the desert hypothesis. In the minimal supersymmetric standard model (MSSM) three gauge couplings meet within the experimental error bound at $M_{\rm GUT} = 2 \times 10^{16}$ GeV with a TeV supersymmetry breaking scale [1, 2]. This scenario involves large logarithms necessary to make the observed α_3 large and $\sin^2\theta_W \simeq 0.231$ at M_Z starting from the universal coupling at high energy, around $\alpha_{3,2,1} \simeq 1/25$ and $\sin^2\theta_W^0 = 3/8$. The wide desert is unavoidable in this scenario and introduces the gauge hierarchy problem [3].

There are other proposals for the solution of the gauge hierarchy problem. The recent effort with the low energy (TeV scale) fundamental scale with extra dimensions tries to solve it simply by abolishing the large hierarchy [4]. If, the fundamental scale is indeed TeV scale, the TeV scale quantum gravity can occur, which is very interesting by itself. Then the conventional supersymmetric grand unified theory (SUSY GUT) should be modified in this framework. Running of gauge couplings in the extra dimensional model is different from the four-dimensional one [5] and reflects several interesting features. With the extra dimension(s), the compactification down to four-dimensional space-time(4D) is a necessity. Starting with odd space-time dimensions, there does not arise the gauge anomaly problem simply because the spinor representations are real. However, the compactification procedure produce complex fermions. Without some twist of the internal space as in the simple torus compactification, the Kaluza-Klein (KK) levels are left-right symmetric, i.e. the fermion representation is vectorlike and there are no massless fermions in the low energy 4D theory. Thus, to have chiral fermions, we should twist the internal space [6, 7]. If the torus is modded out by a discrete group Z_N, then there appear some fixed points. The level matching in the bulk is shifted and the fermions in the bulk need not be vectorlike as in the models studied in refs. [8, 9, 10]. Then, there should be chiral fermions at the fixed points so that there is no anomaly in the effective 4D theory. However, there can be fixed point anomalies [8]. We try to investigate the possibility of TeV scale unification of gauge coupling constants along this line.

Note that the SU(5) unification starts from a large $\sin^2\theta_W^0$ at the unification scale, which needed a wide desert. For a TeV scale unificationin, i.e. in models without desert, therefore, the bare $\sin^2\theta_W^0 = 1/[1+C^2]$ must be close to 0.231, where C^2 defines a properly normalized $\mathrm{U}(1)_Y$ hypercharge. In SU(5), $C^2 = 5/3$ and $\sin^2\theta_W^0 = 3/8$. If $C^2 = 3$, then $\sin^2\theta_W^0 = 1/4$ which can allow TeV scale unification. This $\mathrm{U}(1)_Y$ normalization occurs if $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ is unified in $\mathrm{SU}(3)_W$ with the simplest embedding of $\mathbf{2}_{1/2}$ of $\mathrm{SU}(2)_L\times\mathrm{U}(1)$ into $\mathbf{3}$ of $\mathrm{SU}(3)_W$ [11]. Though leptons and Higgs can be nicely embedded into $\mathrm{SU}(3)_W$, we encounter a difficulty of explaining "fractional" hypercharges of quarks (in the unit of 1/2). Suppose $L = (\nu, e)$ and e^c belong to $\mathbf{3}$ of $\mathrm{SU}(3)_W$. The hypercharges of L and e^c are normalized to $-1/2$ and 1, respectively, and satisfies $\mathrm{Tr}\,Y = 0$ for the $\mathbf{3}$ of $\mathrm{SU}(3)_W$. If $Q = (u, d)$ and d^c come from $\mathbf{3}$ of $\mathrm{SU}(3)_W$, then $Y_Q = 1/6$ and $Y_{d^c} = 1/3$ need an extra U(1) group beyond $\mathrm{SU}(3)_W$, in conflict with our motivation for predicting the weak mixing angle from a unified theory.

Recently five-dimensional(5D) models for gauge coupling unification showed that incomplete multiplets can be consistently introduced at the fixed point (SM fixed point) where only part of the gauge symmetry survives after the orbifold breaking of the gauge symmetry. Though non-universal gauge kinetic terms are introduced in this case, the uncertainty coming from them at the fixed point is suppressed by the large volume factor compared to the universal kinetic term in the bulk. Therefore, we obtain a reasonable unification relation for the low energy gauge coupling constants as long as the volume of the extra dimension is large enough. From the minimally deconstructed point of view, this model can be interpreted as gauge group $G_{\mathrm{GUT}}\times G_{\mathrm{SM\ High}}$ where G_{GUT} is the bulk gauge group and $G_{\mathrm{SM\ High}}$ is the gauge group at the concerned fixed point. Largeness of extra dimension is interpreted as the strong coupling of G_{SM} since low energy gauge coupling after the breaking of the gauge group to its diagonal $G_{\mathrm{SM}} = \mathrm{diag}(G_{\mathrm{GUT}}\oplus G_{\mathrm{SM\ High}})$ is given by

$$\frac{1}{g_i^2} = \frac{1}{g_{\mathrm{GUT}}^2} + \frac{1}{g_{\mathrm{SM\ High},i}^2} \tag{1.1}$$

for the factor group G_i of G_{SM}. Thus in this setup, the difficulty of obtaining correct Y_Q can be easily avoided once we assume that the quark multiplets live at the SM fixed point. Recently there appeared series of papers allowing TeV scale unification of the electroweak sector into $\mathrm{SU}(3)_W$ [12]–[16].

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Now it would be meaningful to search for the possibility of unification in various setups even though there are uncertainties due to the absence of exact spectrum of Higgs and/or superpartners.

It was shown that two gauge coupings meet if the inverse of the extra dimension size is 1–2 TeV for the SM matters (with one Higgs doublet). For supersymmetric case, the size of the extra dimension is given by 3 TeV to 6 TeV [14], and strong coupling scale in which two couplings meet is two orders higher and is about a few hundred TeV. The supersymmetric model in [14] has two Higgses in the bulk and matter multiplets on the fixed point. SUSY breaking scale is assumed to vary from M_Z to 1 TeV and the result is quite sensitive to the spectrum of superpartners. However, putting the matter fields in the bulk or at the fixed points is in a sense arbitrary.

The arbitrariness of putting the matter fields is a bit regulated if we adopt a "naturalness" scheme: if a GUT group is broken at a fixed point, then put a multiplet at that fixed point allowed by the representation of the fixed point gauge group [17]. This was known in string orbifold compactifications [18]. Mainly the fixed point gauge group determines the configuration of the model. One such example was given in ref. [10] in which the SU(5) gauge fields propagates in the bulk. The minimal setup for Higgs is to put H_d in the bulk. The orbifold compactification leads to a chiral fermion in the bulk. So with this representation the effective 4D theory is anomalous. One can introduce H_u also in the bulk. Then the theory is vectorlike and there is no anomaly problem [19]. We call this *vectorlike models*. On the other hand we can put H_u at the fixed point A. The fixed point A has the gauge symmetry $G_{\rm SM} \equiv {\rm SU}(3) \times {\rm SU}(2) \times {\rm U}(1)$ and $G_{\rm SM}$ representations are allowed at the asymmetric fixed point A: some chiral fermions and H_u. This kind of asymmetric embedding of chiral fermions are called *chiral models.*

Similarly with the spirit of ref. [10], we study the ${\rm SU}(3)_W$ models along this line in this paper. In section 2, we present possible ${\rm SU}(3)_W$ models and calculate the running of the gauge coupling constants. In section 3 the proton decay problem is addressed and solved by putting leptons at the *symmetric* fixed point O and quarks at the *asymmetric* fixed point A. In section 4 the neutrino masses are discussed. In section 5 we show that fermionic anomalies at the fixed points are cured by the Chern-Simons term in the bulk in these asymmetric models. Section 6 is a conclusion.

2. Models

The natural assignment of the representations at the fixed points allows only the representations of the fixed point gauge group, but not necessarily the full representation of the unbroken gauge group. We define the *minimal model* in which the fermion representation is put at the fixed point where the gauge group is maximum. Let us consider 5D ${\rm SU}(3)_W$ models.

In the bulk, the gauge fields propagate. In the $S_1/Z_2 \times Z_2'$ compactification with the Sherk-Schwarz mechanism, there appear two fixed points

$$\begin{aligned} O_{\rm GUT}: &\quad y = 0\,, \\ A: &\quad y = \frac{\pi}{2} R\,, \end{aligned}$$

where O is the *symmetric fixed point* where $\mathrm{SU}(3)_W$ is not broken, and A is the *asymmetric fixed point* where SU(3) is broken down to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Let us put fermions at the fixed points in the minimal scheme. The leptons L and e^c can form an $\mathrm{SU}(3)_W$ triplet $\mathbf{3}$, and hence we put it at the symmetric fixed point O. On the other hand, Q, u^c and d^c cannot form $\mathrm{SU}(3)_W$ representations. So they must be put at the asymmetric fixed point A. The Higgs doublet $H_d(Y=-1/2)$ or $H_u(Y=1/2)$ can be assigned into $\mathbf{3}_H$ or $\bar{\mathbf{3}}_H$ of $\mathrm{SU}(3)_W$. Hence, if the vacuum expectation value of $\mathbf{3}_H$ gives mass to electron, it must be put where $\mathrm{SU}(3)_W$ is a good symmetry, i.e. in the bulk or at O. If this $\mathbf{3}_H$ also gives mass to quark(s), then it must be put in the bulk. Since it is more economic to have less Higgs fields, the minimal model dictates to put $\mathbf{3}_H$ in the bulk. On the other hand, we put the Higgs doublet H_u at the asymmetric fixed point A.

This minimal setup has several merits. First, rapid proton decay can be avoided if the size of the extra dimension is slightly bigger than the fundamental length since quarks and leptons are located at the opposite fixed points and the locality of the extra dimension prevents the contact term giving rapid proton decay. Second, it gives a geometric explanation of $b-t$ mass ratio [10].

With supersymmetry, there is another possibility for the fixed point gauge symmetry *a la* the Sherk-Schwarz mechanism as discussed in [14, section 4]. When we compactify 5D on an $S^1/(Z_2\times Z_2')$, Z_2 and Z_2' turn out to be equivalent to one orbifolding Z for $y\to -y$ and one twist T for $y\to y+\pi R$. So we conveniently write the boundary conditions of the gauge field $A_M=(A_\mu,A_5)$ $(\mu=0,1,2,3)$ in terms of Z and T:

$$\begin{aligned} A_\mu(y) &= ZA_\mu(-y)Z^{-1} = TA_\mu(y+\pi R)T^{-1}\,, \\ A_5(y) &= -ZA_5(-y)Z^{-1} = TA_5(y+\pi R)T^{-1}\,, \end{aligned} \tag{2.1}$$

where Z, T and $A_M = A_M^a T^a$ are represented by 3×3 matrices. Then there are two independent choices for Z,T for breaking $\mathrm{SU}(3)_W$ into $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$,

$$\begin{aligned} \text{Type I}: &\quad (Z,T) = (\mathrm{diag}(1,1,1),\mathrm{diag}(1,1,-1))\,, \\ \text{Type II}: &\quad (Z,T) = (\mathrm{diag}(1,1,-1),\mathrm{diag}(1,1,-1))\,. \end{aligned}$$

For the former case, the $\mathrm{SU}(3)_W$ symmetry breaks into $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ only at the fixed point A while it is fully conserved at the fixed point O. On the other hand, for the latter case, the $\mathrm{SU}(3)_W$ symmetry is broken into $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ at both fixed points. However, for type-II, there also appears a massless adjoint scalar of $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$, which nonetheless gets a radiative mass of the order of the compactification scale.

Furthermore, when we introduce a Higgs triplet $\mathbf{3}_H=(H_D,H_S)$ including a doublet with $Y=-1/2$ and a singlet with $Y=1$ in the bulk, only the Higgs doublet remains massless by the boundary conditions with the same (Z,T) as for the gauge field:

$$\mathbf{3}_H(y) = Z\mathbf{3}_H(-y) = T\mathbf{3}_H(y+\pi R)\,. \tag{2.2}$$

But, for type-I, when the lepton triplet resides at the fixed point O where $\mathrm{SU}(3)_W$ is fully operative, it is impossible to have realistic charged lepton masses with a Higgs triplet via

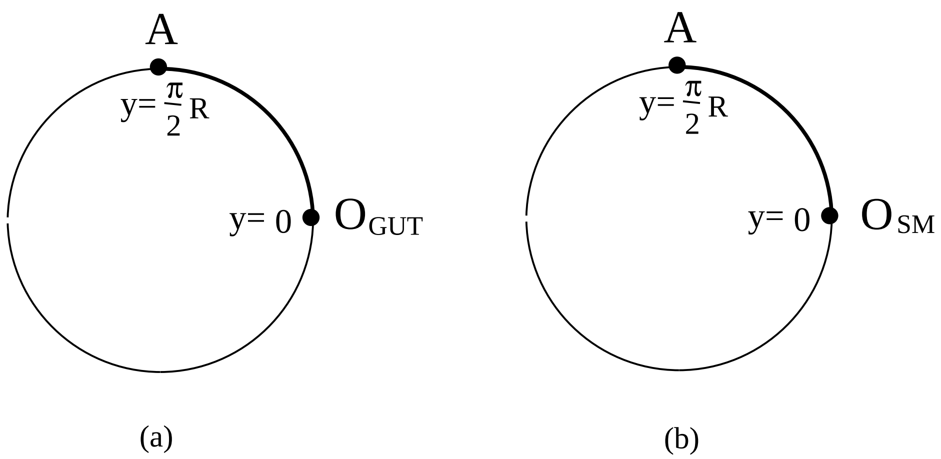

Figure 1: The asymmetric point A preserves only the SM gauge group. On the other hand, the symmetric point O can (a) preserve $\mathrm{SU}(3)_W$, or preserve only(b) the SM gauge group, which are explicitly shown as $O_{\rm GUT}$ and $O_{\rm SM}$, respectively. In this paper, we consider only $O_{\rm GUT}$.

$\mathbf{3}_\ell\mathbf{3}_\ell\mathbf{3}_H$. Thus, in that case, we instead should introduce a Higgs sextet $\bar{\mathbf{6}}_H = (H_D, H_T, H_S)$ including a doublet with $Y = -1/2$, a triplet with $Y = 1$ and a singlet with $Y = -2$ satisfying the boundary conditions as

$$\bar{\mathbf{6}}_H(y) = Z\bar{\mathbf{6}}_H(-y)Z^{-1} = T\bar{\mathbf{6}}_H(y+\pi R)T^{-1} \,. \tag{2.3}$$

The fixed point O also breaks $\mathrm{SU}(3)_W$ down to $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. We call it $O_{\rm SM}$ in this case.

We distinguish these two cases by

$$\begin{aligned} \text{Type I}: &\quad O_{\rm GUT} \text{ and A}\,, \\ \text{Type II}: &\quad O_{\rm SM} \text{ and A}\,. \end{aligned} \tag{2.4}$$

Even in type-II models, quarks should be put at A since quarks can not fit to the hypercharges of the triplet. Leptons form $\mathbf{3}$ of $\mathrm{SU}(3)_W$ and can be located at O. The schematic behavior of the symmetries at O is shown in figure 1.

In any case, the minimal model with supersymmetry dictates to put the fields as follows,

$$\begin{aligned} O(y=0): &\quad L, e^c\,, \\ \mathrm{B(ulk)}: &\quad A_M, H_d\,, \qquad \text{(with supersymmetry)}\,, \\ A\left(y=\frac{\pi}{2}R\right): &\quad Q, u^c, d^c, H_u\,. \end{aligned} \tag{2.5}$$

For non-supersymmetric case, one Higgs is enough for the generation of masses in its minimal form and two Higgs doublet (2HD) model involves second Higgs. However, for supersymmetric case, two Higgses are necessary but the necessary positions are different. At O, we need H_d to give masses to charged leptons. At A, we need H_u and H_d to give masses to up-type and down-type quarks. From this, we can conclude that H_u is necessary at A and H_d is necessary at O and A. Therefore, the minimal setup is to have H_u at A and H_d in the bulk.

In this paper, therefore, we study following models,

Model A: Scherk-Schwarz gauge symmetry breaking without supersymmetry

We consider type-I with $O_{\rm GUT}$ and A for non-supersymmetric cases. First, one Higgs model (A-1) is summarized which has been studied already. Two Higgs doublet Standard Model is realized in 5D as a vectorlike model (A-2v) and a chiral model (A-2c).

Model B: Scherk-Schwarz gauge symmetry breaking with supersymmetry

At the scale $1/R$, the gauge symmetry is broken but $N=1$ supersymmetry is unbroken.[1] Supersymmetry requires two Higgs doublets. We review vectorlike realization (B-v) and chiral realization (B-c) of MSSM in 5D. In general, the bulk supersymmetry with eight supercharges restricts possible interactions of the theory and the model is highly predictive except the Fayet-Iliopoulos(FI) term allowed at the fixed points. The FI term is

$$\mathcal{L}_{FI} = \xi_1 D\delta(y) + \xi_2 D\delta\left(y - \frac{\pi R}{2}\right). \tag{2.6}$$

If $N=1$ is unbroken, the integrated FI term ($(\xi_1+\xi_2)D$) does not get any radiative correction for $\mathrm{U}(1)_Y$ since the FI term is generated only for anomalous U(1)'s. Therefore, $\xi_1+\xi_2=0$ is a stable setup from the naturalness argument. For type-I, there is no U(1) at $O_{\rm GUT}$ and $\xi_1=0$. Therefore, combining two facts (unbroken non-abelian gauge group at one fixed point $O_{\rm GUT}$ and 4D $N=1$ supersymmetry) tells us that the absence of FI term at A ($\xi_2=0$) is stable against radiative corrections. This is one of the merits which allow the analysis simple. For type-II, quadratically divergent FI terms are generated at both fixed points with opposite sign and this makes the configuration complicated and unpredictable[2] Though the physics related to the Fayet-Iliopoulos term is interesting by itself, it goes beyond the scope of this paper and we consider only type-I in the paper.

Model C: Scherk-Schwarz breaking of both gauge symmetry and supersymmetry

Here, the compactification breaks gauge symmetry and supersymmetry at the same time. If supersymmetry is also broken by Scherk-Schwarz mechanism, the Fayet-Iliopoulos term is generated at A even for type-I since the supersymmetric condition $\xi_1+\xi_2=0$ for one-loop correction disappears and $\xi_1=0$ cannot determine the stable value for ξ_2. This makes the analysis much more complicated and we do not consider Model C in this paper.

Running of gauge couplings is given by [22]

$$\begin{aligned}
\frac{1}{g^2(M_Z)} &= \frac{1}{g_3^2} - \frac{b_g}{8\pi^2}\log\frac{M_{c'}}{M_Z} - \frac{\tilde{b}_g}{8\pi^2}\log\frac{M_s}{M_{c'}}\,, \\
\frac{1}{{g'}^2(M_Z)} &= \frac{1}{g_3^2} - \frac{b_{g'}}{8\pi^2}\log\frac{M_{c'}}{M_Z} - \frac{\tilde{b}_{g'}}{8\pi^2}\log\frac{M_s}{M_{c'}}\,.
\end{aligned}$$

[1]Supersymmetry is assumed to be broken at M_Z or 1 TeV at one fixed point and gaugino mediates supersymmetry breaking to the opposite (or SM) fixed point. If leptons are at the fixed point w here supersymmetry is broken, flavor changing neutral currects are induced generally. Thus, phenomenologically, all the SM matter fields are required to be at A if supersymmetry is broken at O.

[2]Localization of bulk fields by the VEV of (parity-odd) adjoint chiral fields [20] and correction to the gauge coupling running through Chern-Simons term [21] are two major effects of FI terms.

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Here we neglect non-universal brane gauge kinetic terms by the strong coupling assumption and the (moderate) largeness of the extra dimension. Strong coupling behavior of 5-D gauge coupling fixes the ratio $M_s/M_{c'}$ to be $\mathcal{O}(100)$ (100 or $16\pi^3 \sim 500$).

It is convenient for later uses to define the relevant combination of the beta functions in determining the unification scale.

$$B = b_g - \frac{b_{g'}}{3}, \qquad \tilde{B} = \tilde{b}_g - \frac{\tilde{b}_{g'}}{3}.$$

Model A-1: standard model (SM) with one Higgs doublet. The beta functions below the compactification scale are $b_g = 19/6$, $b_{g'} = -41/6$ and $B = 49/9$. Above the compactification scale, the beta functions are modified: $\tilde{b}_g = -1/4$, $\tilde{b}_{g'} = -27/4$ and $\tilde{B} = 2$. Useful formula is [14]

$$\begin{aligned}\sin^2\theta_W &= \frac{1}{4} - \frac{3}{8\pi}\alpha_{\rm em}\left[\left(\tilde{b}_g - \frac{\tilde{b}_{g'}}{3}\right)\log\frac{M_s}{M_{c'}} + \left(b_g - \frac{b_{g'}}{3}\right)\log\frac{M_{c'}}{M_Z}\right] \\ &= \frac{1}{4} - \frac{3}{8\pi}\alpha_{\rm em}\left[\tilde{B}\log\frac{M_s}{M_{c'}} + B\log\frac{M_{c'}}{M_Z}\right].\end{aligned}$$

From this formula, once we fix $(M_s/M_{c'}) \approx 100$ or $16\pi^3$ from naive dimensional analysis, following strong coupling assumption, we can determine $M_{c'}$ by putting beta function coefficients for different models. For $(M_s/M_{c'}) = 100(16\pi^3)$, the unification scale is given as 70(190) TeV which is just two to three orders higher compared to the electroweak scale.

Model A-2v: two Higgs doublet SM with vectorlike embedding. The beta functions below the compactification scale are $b_g = 3$, $b_{g'} = -21/3$ and $B = 16/3$. Above $1/R$, $\tilde{b}_g = (23/6, -1/6, -4)$ for gauge, Higgs and matters, respectively, and $\tilde{b}_{g'} = (0, -1/6, -20/3)$. Total sum is $\tilde{b}_g = -(1/3)$, $\tilde{b}_{g'} = -(41/6)$ and $\tilde{B} = 35/18$. Then we can determine the unification scale M_s and it is 80(210) TeV for $M_s/M_{c'} = 100(16\pi^3)$.

Model A-2c: two Higgs doublet SM with chiral embedding. This is the minimal model for Higgs configuration. From the minimality condition, leptons are at $O_{\rm GUT}$ and quarks are at A, then H_d should be in the bulk and H_u should be at A for the two Higgs doublet SM in which both Higgses play roles in giving fermion masses. Below $1/R$, the beta function is the same as Model A-2v. Above $1/R$, $\tilde{b}_g = (23/6, -1/4, -4)$ for gauge, Higgs and matters, respectively, and $\tilde{b}_{g'} = (0, -1/4, -20/3)$. The sum is $\tilde{b}_g = -5/12$, $\tilde{b}_{g'} = -83/12$ and $\tilde{B} = 17/9$. The unification scale is then 80(220) TeV for $M_s/M_{c'} = 100(16\pi^3)$.

Model B-v: MSSM with vectorlike embedding. In this model both H_d and H_u are put in the bulk. Phenomenological constraint requires H_d from $\bar{\mathbf{6}}_H$ rather than $\mathbf{3}_H$ to give correct charged lepton masses. However, the contribution of $\bar{\mathbf{6}}_H$ and $\mathbf{3}_H$ to the beta function is the same. The reason is following. The members of the hypermultiplet $\bar{\mathbf{6}}_H = \{H_D, H_T, H_S, \hat{H}_S, \hat{H}_T, \hat{H}_D\}$ are assigned the $Z_2 \times Z_2'$ parity as $\{(++), (+-), (+-), (-+), (-+), (--)\}$. Likewise, for $\mathbf{3}_H = \{H_D, H_S, \hat{H}_S, \hat{H}_D\}$, the parity assignments are $\{(++), (+-), (-+), (--)\}$. Since only the even modes with $(++)$ and $(--)$ contribute to the logarithmic running, $\bar{\mathbf{6}}_H$ and $\mathbf{3}_H$ give the same contribution to the beta function.

The running interval for the supersymmetric models can be divided into three parts. From M_Z to $M_{\rm SUSY}$, the running is governed by SM. From $M_{\rm SUSY}$ to $M_{c'}$, the running is that of the usual MSSM. From $M_{c'}$ to M_s, we follow the analysis given in [10]. Though $M_{\rm SUSY}$ can vary from M_Z to 1 TeV, we identify $M_{\rm SUSY}$ with M_Z for the simple analysis. Dependence of the unification scale on the detailed sparticle spectrum is quite strong and the result should be understood as a qualitative one with uncertainties from underdetermined sparticle spectrum. The beta function coefficients are the following. Below $1/R$, $b_g = -1, b_{g'} = -11$ and $B = 8/3$. Above the compactification scale, $\tilde{b}_g = (4, 0, -6)$ for gauge, Higgs and matters, respectively, and $\tilde{b}_{g'} = (0, 0, -10)$. The sum is $\tilde{b}_g = -2$, $\tilde{b}_{g'} = -10$ and $\tilde{B} = 4/3$. The unification scale is determined to be $1.9(4.2) \times 10^4$ TeV for $M_s/M_{c'} = 100(16\pi^3)$. This unification scale becomes slightly lower if we change $M_{\rm SUSY}$ to 1 TeV from M_Z but is still highler compared to the non-supersymmetric models.

Model B-c (minimal model): MSSM with chiral embedding. In this model, H_d is put in the bulk and H_u is put at the fixed point A. From the minimality condition, leptons are at $O_{\rm GUT}$ and quarks are at A. H_d should couple at both fixed points in order to give mass to charged leptons and down-type quarks and H_u is necessary only at A. Therefore, Model B-c realizes the minimal configuration required by the phenomenological constraint. The running below $1/R$ is the same as Model B-v. Above the compactification scale, $\tilde{b}_g = (4, 0, -13/2)$ for gauge, Higgs and matters, respectively, and $\tilde{b}_{g'} = (0, 0, -21/2)$. $\tilde{b}_g = -5/2, \tilde{b}_{g'} = -21/2$ and $\tilde{B} = 1$. Now the unification scale is a little bit higher than the Model B-v, $M_s = 3.4(9.2) \times 10^4$ TeV for $M_s/M_{c'} = 100(16\pi^3)$.

In the previous analysis, the threshold corrrections are neglected and the possible effects from non-universal brane kinetic terms at A are assumed to vanish. The suppression of the brane kinetic term is justified by the strong coupling assumption once the size of the extra dimension is larger than the Planck length (or string length $l_s = 1/M_s$).

3. Proton decay

Longevity of proton at this moment gives strong constraint on the models with low fundamental scale. Extremely many operators consistent with the symmetries of the theory should be forbidden in order to avoid rapid proton decay for low fundamental scale models. Though it is possible to forbid all these unwanted B violating operators by imposing discrete gauge symmetry, the geometric explanation for the suppression of B violating operators [23] is much simpler and nicer. Once the quarks and leptons are at different positions along the extra dimension, the B violating operators are forbidden by the locality of higher-dimensional field theory. Non-local B violating terms are generated by non-perturbative or quantum gravitational effects at low energy but in the models considered above it is suppressed by the separated length as $e^{-M_s/M_{c'}} \sim e^{-100} \sim 10^{-44}$ since quarks and leptons are maximally separated in the extra dimension. Exponential suppression of unwanted B violating operators is easily obtained once we separate quarks and leptons a few tens

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times the fundamental length scale. The minimality condition of $SU(3)_W$ puts quarks at A and leptons at $O_{\rm GUT}$ and the proton decay is naturally avoided by the minimality condition.[3]

4. Neutrino mass

In order to give mass to neutrinos, H_u also should act at O. Thus now we have to consider two bulk Higgses. However,

$$\begin{aligned}\mathcal{L} &= \frac{\lambda}{M_*} 3_L 3_L \bar{3}_{H_u} \bar{3}_{H_u} \delta(y) \\ &= \frac{\lambda}{M_*} LLH_uH_u\delta(y) + \cdots \end{aligned} \tag{4.1}$$

gives too large masses to neutrinos unless M_* is extremely higher than the electroweak scale ($\sim \langle H_u \rangle$). By giving large kink mass we can make H_u almost localize at A and this explains the smallness of neutrino masses through the exponential suppresion of H_u wave function. The detailed realization for the H_u localization needs extra modification of the models. $U(1)_Y$ cannot play an asymmetric localization since the hypercharge of H_u and H_d is exactly opposite and if H_u is localized at one fixed point and then H_d is localized at the other fixed point. Therefore, we should introduce anomalous $U(1)_A$ under which H_u and H_d are asymmetrically charged. (For instance, only H_u is charged under $U(1)_A$.)

5. Fixed point anomaly cancellation

Since we deal with chiral models, it is necessary to justify how the cancellation of the local gauge anomaly at the orbifold fixed point is realized in detail.

Anomaly is an IR (infrared) property of the theory and is determined from the massless sector of the model in 4D. However, there can exist a local gauge anomaly spread along the extra dimension if we spread quarks and leptons at different fixed points even when the integrated anomaly cancels in the effective 4D theory. Therefore, it is necessary to check whether the fixed point gauge anomaly can be cancelled by writing down local counter terms.

Firstly, let us briefly review the result on the abelian gauge anomaly. For a 5D orbifold, the local gauge anomaly appears only at the fixed point and is equally distributed for an abelian gauge group. For $U(1)$ gauge group with one unit-charged fermion in the bulk on an $\mathbb{S}^1/\mathbb{Z}_2$ orbifold, the local gauge anomaly is [8],

$$\partial_M J^M(x,y) = \frac{1}{2}\left[\delta(y) + \delta(y-\pi R)\right] \mathcal{Q} \,, \tag{5.1}$$

[3] More precisely, leptons can be placed in the bulk and can have contact interations at A with quarks. Therefore, we should assume that matter chiral fields live only at fixed points. In this case, different locations of leptons and quarks are derived from the minimality condition. The presence of anomalous $U(1)$ under which leptons are charged allows leptons to be localized at the fixed point via Fayet-Iliopoulos term. However, in this case the separation of leptons and quarks are not guaranteed.

where J^M is the five-dimensional current and

$$\mathcal{Q} = \frac{1}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \tag{5.2}$$

is the four-dimensional chiral anomaly. Starting from a theory with bulk fermions, we can calculate the contributions of all the Kaluza-Klein modes to the anomaly and the answer is independent of the wave functions. The chiral fermion contribution localized at the fixed point is

$$\partial_\mu J^\mu(x,y) = \delta(y)\, \mathcal{Q}\,, \tag{5.3}$$

and the Chern-Simons term contribution is

$$\partial_5 J^5(x,y) = \frac{1}{2} \left[-\delta(y) + \delta(y - \pi R)\right] \mathcal{Q} \tag{5.4}$$

from

$$\mathcal{L}_{\rm CS} = -\frac{1}{128\pi^2} \epsilon(y) \epsilon_{MNPQR} A^M F^{NP} F^{QR}\,. \tag{5.5}$$

where $\epsilon(y) = \pm 1$ for $y > 0$ and $y < 0$, respectively.

The above calculation can be extended to $S^1/(Z_2 \times Z_2^{'})$ [24]. Since the five-dimensional Dirac spinor has left- and right-handed spinors, ψ_L, ψ_R, from the 4D point of view, there are two possibilities for the parity assignment of the fermions,

Case (1) ψ_L: $(+,+)$, ψ_R: $(-,-)$

Case (2) ψ_L: $(+,-)$, ψ_R: $(-,+)$

For Case (1) with the parity $(+,+)$ and $(-,-)$, the anomaly calculation is the same as the previous result and the local gauge anomaly is

$$\partial_M J^M(x,y) = \frac{1}{2} \left[\delta(y) + \delta\left(y - \frac{\pi R}{2}\right)\right] \mathcal{Q}\,. \tag{5.6}$$

For the second case with $(+,-)$ and $(-,+)$, the flip of the parity under $Z_2^{'}$ can be represented by the twisting of the corresponding fields under the translation $y \to y + \pi R$ for the corresponding wave functions. Therefore, the answer is [24]

$$\partial_M J^M(x,y) = \frac{1}{2} \left[\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right] \mathcal{Q}\,. \tag{5.7}$$

It is easy to see that this local gauge anomaly can be cancelled exactly by the bulk Chern-Simons term,

$$\mathcal{L}_{\rm CS} = -\frac{1}{128\pi^2} \epsilon(y) \epsilon_{MNPQR} A^M F^{NP} F^{QR}\,. \tag{5.8}$$

Now, it is quite straightforward to generalize the above review on the abelian gauge anomaly cancellation to the non-abelian case.

5.1 Non-abelian anomaly from bulk matters: unbroken case

When the gauge symmetry is not broken, the previous formula is valid. For instance, the anomaly for the fermion N of $\mathrm{SU}(N)$ with the parity $(+,(-1)^{h_i})$ and $(-,-(-1)^{h_i}))$ is

$$D_M J^{Ma} = \frac{1}{2}\left[\delta(y) + (-1)^{h_i}\delta\left(y - \frac{\pi R}{2}\right)\right]\mathcal{Q}^a\,, \tag{5.9}$$

where

$$\mathcal{Q}^a = \frac{1}{32\pi^2}D^{abc}F^b_{\mu\nu}\tilde{F}^{c\mu\nu}\,, \tag{5.10}$$

and

$$D^{abc} = \frac{1}{2}\,\mathrm{tr}\left(\left\{T^a, T^b\right\}T^c\right). \tag{5.11}$$

If the fermion contains massless zero mode ($h_i = 0$), the anomaly appears at both fixed points with equal sign, and if it does not have massless zero mode ($h_i = 1$), the anomaly appears with opposite sign. This is a trivial generalization of the abelian result.

5.2 Non-abelian anomaly cancellation with Scherk-Schwarz breaking of gauge symmetry

However, the most interesting models involve the breaking of gauge symmetry with the parity. Then the symmetric fixed point O_{GUT} and the other fixed point A should be distinguised. For the bulk gauge group G, the Z_2' parity which does not commute with G break the gauge group to $H = H_1 \times H_2 \times \cdots \times H_n$. The anomaly at O_{GUT} is expressed G invariantly and is the same as the above. At A, the expression needs more information.

5.2.1 Bulk fermion contribution

Bulk fermions belonging to the fundamental representation of G is divided into fundamentals under H_i and have different parities under Z_2'. Let the parity be $(+,(-1)^{h_i})$ and $(-,-(-1)^{h_i})$ for ψ_L and ψ_R. Then we can calculate the anomaly induced at A from the knowledge of abelian gauge anomaly just like the previous section except the fact that now $\mathcal{Q}$ is not the anomaly of entire bulk gauge group but is that of unbroken subgroup. Thus the general expression of the anomaly from the bulk matter is

$$D^M J^a_M = \frac{1}{2}\delta(y)\mathcal{Q}^a_O + \frac{1}{2}\delta\left(y - \frac{\pi R}{2}\right)\sum_i (-1)^{h_i}\mathcal{Q}^{a_i}_{Ai}\delta_{aa_i}\,, \tag{5.12}$$

where

$$\mathcal{Q}^a_O = \frac{1}{32\pi^2}D^{abc}F^b_{\mu\nu}\tilde{F}^{c\mu\nu}\,, \tag{5.13}$$

and

$$\mathcal{Q}^{a_i}_{Ai} = \frac{1}{32\pi^2}D^{a_ib_ic_i}F^{b_i}_{\mu\nu}\tilde{F}^{c_i\mu\nu}\,. \tag{5.14}$$

Here, a is the gauge index for unbroken group G and a_i is the gauge index of its subgroup H_i.

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5.2.2 Chern-Simons contribution

The bulk Chern-Simons term is

$$\mathcal{L}_{\rm CS} = -\frac{1}{128\pi^2}\epsilon(y)\,{\rm tr}\left(AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5\right). \tag{5.15}$$

The gauge transformation leaves a non-vanishing term at the boundary which can cancel the anomaly from the chiral fermions. The general expression of the anomaly from the Chern-Simons term is

$$D^M J^a_M = -\frac{1}{2}\delta(y)\,\mathcal{Q}^a_O + \frac{1}{2}\delta\left(y - \frac{\pi R}{2}\right)\sum_i \mathcal{Q}^{a_i}_{Ai}\delta_{aa_i}\,, \tag{5.16}$$

where

$$\mathcal{Q}_O = \frac{1}{32\pi^2}D^{abc}F^b_{\mu\nu}\tilde{F}^{c\mu\nu}\,, \tag{5.17}$$

and

$$\mathcal{Q}_{Ai} = \frac{1}{32\pi^2}D^{a_ib_ic_i}F^{b_i}_{\mu\nu}\tilde{F}^{c_i\mu\nu}\,. \tag{5.18}$$

5.2.3 Bulk fermion, Chern-Simons term, and brane fermion contributions

If we add two contributions, the anomaly at $O_{\rm GUT}$ cancel with each other perfectly. The anomaly at A is

$$D^M J^a_M = \delta\left(y - \frac{\pi R}{2}\right)\sum_i \mathcal{Q}^{a_i}_{Ai}\delta_{aa_i}\delta_{h_i0}\,. \tag{5.19}$$

The contributions of bulk fermions with $h_i = 1$ and Chern-Simons term cancel while the contributions of those with $h_i = 0$ and CS term add up. This is the anomaly that can be cancelled exactly by putting incomplete multiplet associated to the zero mode which is (anti-)fundamental under H_i (or carries opposite charges compared to the zero mode from the bulk fermion). This clearly shows the cancellation mechanism of cubic gauge anomaly like YM-YM-YM, U(1)-YM-YM and U(1)^3.

5.3 U(1) gauge boson-graviton-graviton anomaly cancellation

In the previous sections, we studied the cubic anomaly cancellation of abelian and non-abelian gauge groups. In this subsection, the mixed anomaly for U(1) gauge boson-graviton-graviton is considered. For an abelian gauge group, U(1)-graviton-graviton anomaly cancellation is done in parallel to U(1)^3 anomaly cancellation. At each fixed point the anomaly is

$$\partial_M J^M(x,y) = \frac{1}{2}\left[\delta(y) + \delta\left(y - \frac{\pi R}{2}\right)\right]\mathcal{Q}_{\mathcal{G}}\,, \tag{5.20}$$

where J^M is the five-dimensional current and

$$\mathcal{Q}_{\mathcal{G}} = \frac{1}{192\pi^2}R_{\mu\nu}\tilde{R}^{\mu\nu} \tag{5.21}$$

is the gravitational anomaly. The chiral fermion localized at the fixed point gives

$$\partial_\mu J^\mu(x,y) = \delta(y)\,\mathcal{Q}_{\mathcal{G}} \tag{5.22}$$

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and the Chern-Simons term gives

$$\partial_5 J^5(x,y) = \frac{1}{2}\left[-\delta(y) + \delta\left(y - \frac{\pi R}{2}\right)\right]\mathcal{Q}_{\mathcal{G}}\,, \tag{5.23}$$

where

$$\mathcal{L}_{\rm CS} = -\frac{1}{768\pi^2}\epsilon(y)\epsilon_{MNPQR}A^M R^{NP}R^{QR}\,. \tag{5.24}$$

We obtain a similar conclusion for $S^1/Z_2 \times Z_2'$.

The apparent problem appears when a U(1) gauge group survives after the breaking of the bulk non-abelian gauge group. In this case, the gravitational mixed anomaly of U(1) is induced only at the fixed point A and should be cancelled by localized fields at A since the bulk has non-abelian gauge symmetry and does not allow a gravitational Chern-Simons term like $A \wedge R \wedge R$. To see how this works, let us consider the breaking of $\mathrm{SU}(M+N)$ to $\mathrm{SU}(M)\times$ $\mathrm{SU}(N) \times \mathrm{U}(1)$ by Scherk-Schwarz mechanism at A. If the fundamental $M+N$ of $\mathrm{SU}(M+N)$ is in the bulk has a parity assignment $(+,-)$ and $(-,+)$ for M_{aN} and $(+,+)$ and $(-,-)$ for N_{-aM}, there appears a U(1) gravitational mixed anomaly at A

$$\partial_M J^M(x,y) = \frac{1}{2}\left[(-1)M \times aN + N \times (-aM)\right]\delta\left(y - \frac{\pi R}{2}\right)\mathcal{Q}_{\mathcal{G}}\,, \tag{5.25}$$

where the subscript aN and $-aM$ denote U(1) charge and we haven't fixed the normalization of U(1) charge a. Since the U(1) is a subgroup of simple group, the sum of U(1) charges should vanish. Therefore the anomaly becomes

$$\partial_M J^M(x,y) = N \times (-aM)\delta\left(y - \frac{\pi R}{2}\right)\mathcal{Q}_{\mathcal{G}}\,, \tag{5.26}$$

and is cancelled by the U(1)-graviton-graviton anomaly induced from $\bar{N}_{aM}$ localized at A. This result is very interesting since we cannot write down YM-gravity-gravity mixed Chern-Simons term $A\wedge R\wedge R$ due to $\mathrm{Trace}(A) = 0$. The U(1) gravitational mixed anomaly should be cancelled without the aid of Chern-Simons term and this is the case indeed.

This is in accord with the absence of the Fayet-Iliopoulos term in the setup. $N = 1$ supersymmetry relates U(1) gravitational mixed anomaly from the fermions to the FI term from the bosons, and the absence of the anomaly guarantees the absence of quadratically divergent FI term at the fixed point. Therefore, the vanishing of FI term is natural and is protected from radiative corrections.

5.4 SU(5)

Even though we are studying the $\mathrm{SU}(3)_W$ model here, it is appropriate to see the cancellation of the anomalies for the chiral model [10] since we presented above the general mechanism for the fixed point anomaly cancellation. In the bulk there exists one **5** with parity $(-,-,-,+,+)$ and one doublet **2** with opposite hypercharge is locate at A. H_d is the zero mode coming from **5** and is $\mathbf{2}_{-1/2}$. H_u located at A is $\mathbf{2}_{1/2}$. The cubic anomaly of SU(5) from **5** appears at O and should be cancelled by the Chern-Simons term. Then at A, the cubic anomaly of the subgroup $\mathrm{SU}(3)_C^3$ and $\mathrm{U}(1)_Y^3$ (and also the mixed anomaly

$\mathrm{U}(1)_Y$-$\mathrm{SU}(3)_C$-$\mathrm{SU}(3)_C$ from the triplet of **5** and the Chern-Simons term cancel with each other. The cubic anomaly $\mathrm{SU}(2)_L^3$ and $\mathrm{U}(1)_Y^3$ (and the mixed anomaly $\mathrm{U}(1)_Y$-$\mathrm{SU}(2)_L$-$\mathrm{SU}(2)_L$) from the doublet of 5 is added with the one from the Chern-Simons term and is cancelled by the anomaly from the doublet living at A. The gravitational mixed anomaly of U(1) from **5** can appear at A as

$$\begin{aligned}\partial_M J^M(x,y) &= \frac{1}{2}\left[(-1)\times 3\times\frac{1}{3}+2\times\left(-\frac{1}{2}\right)\right]\delta\left(y-\frac{\pi R}{2}\right)\mathcal{Q}_{\mathcal{G}} \\ &= (-1)\delta(y-\pi R)\mathcal{Q}_{\mathcal{G}}\,, \end{aligned} \tag{5.27}$$

where the first term is from $\mathbf{3}_{1/3}$ and the second is from $\mathbf{2}_{-1/2}$. This is exactly cancelled by $\mathrm{U}(1)_Y$-gravity-gravity anomaly from $\mathbf{2}_{1/2}$ living at A.

5.5 $\mathrm{SU}(3)_W$

The anomaly of Higgs sector cancels independently and quark-lepton sector cancels with appropriate Chern-Simons terms. In the previous discussions it was proven that once the 4D gauge anomaly cancel the 5D fixed point gauge anomaly can be cancelled with having appropriate Chern-Simions term for the Scherk-Schwarz breaking setup of gauge symmetry.

Considering $T_8 = -Y/\sqrt{3}$, we find that the fixed point anomalies from leptons $\mathbf{3}_l$ appear only at O_{GUT} and are cancelled by the 5D Chern-Simons term,

$$\mathcal{L}_{\mathrm{CS}} = -\frac{c_l}{128\pi^2}\epsilon(y)\,\mathrm{tr}(A\wedge F\wedge F-\cdots)\,, \tag{5.28}$$

by choosing $c_l = 2$ ($3c_l = 6$ for three generations) since the anomaly from the fixed point localized matter is twice of that of bulk matter. This Chern-Simons term at the same time cancel the anomalies at A from quarks. Moreover, for the supersymmetric $\mathrm{SU}(3)_W$ case, the anomaly from the bulk Higgsino $\bar{\mathbf{6}}_H$ is the same as that of **3** and appear at O_{GUT} and A. The anomaly at O_{GUT} from $\bar{\mathbf{6}}_H$ is cancelled by the Chern-Simons term with $c_H = 1$, and the anomalies at A from $\bar{\mathbf{6}}_H$ and the Chern-Simons term are cancelled by the contribution from the Higgs doublet $\mathbf{2}_{H_u}$ at A. Thus, if $c = 3c_l + c_H$ is chosen to be 7, all the fixed point anomaly from quarks, leptons and Higgsinos disappear. For the model in which both Higgses are in the bulk, $c = 3c_l = 6$ is enough to cancel all the fixed point anomalies.

Gravitational mixed anomaly of $\mathrm{U}(1)_Y$ appears only at A. Quarks do not give the anomaly since $\sum Y_{q_i} = 0$. $\bar{\mathbf{6}}_H$ gives the anomaly at A as

$$\begin{aligned}\partial_M J^M(x,y) &= \frac{1}{2}\left[(-1)\times 1\times(-2)+2\times\left(-\frac{1}{2}\right)+(-1)\times 3\times 1\right]\delta\left(y-\frac{\pi R}{2}\right)\mathcal{Q}_{\mathcal{G}} \\ &= (-1)\delta(y-\pi R)\mathcal{Q}_{\mathcal{G}}\,, \end{aligned} \tag{5.29}$$

where the first term is from $\mathbf{1}_{-2}$ and the second is from $\mathbf{2}_{-1/2}$ and the third term is from $\mathbf{3}_1$. Notice (-1) factors for the singlet and the triplet due to the parity assignment. This is just twice of the bulk doublet contribution and is exactly cancelled by $\mathrm{U}(1)_Y$-gravity-gravity anomaly from $\mathbf{2}_{1/2}$ living at A.

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6. Conclusion

In this paper we examined the possibility of unification of electroweak gauge group into $SU(3)_W$ with minimality condition. For the simplest setup, the tree level prediction is very close to the observed value and the unification is achieved near TeV. As a specific model, "natural" location of matters are assumed such that leptons are at O and quarks are at A. The minimal configuration requires H_d at both fixed points but H_u only at one fixed point A if we neglect neutrino masses. The setup can avoid proton decay and can explain the bottom-top mass ratio with order one $\tan\beta$ as a byproduct of the minimality condition. Neutrino masses are easily incorporated once we put H_u also in the bulk. Once kink mass of H_u is given, then H_u is almost localized at A and neutrinos acquire tiny mass even for a very low fundamental scale. The vectorlike and chiral models we considered in this paper are shown to be made free of anomalies even at the fixed points. From this experience, we conjecture that the orbifold compactification can be made sensible even at the fixed points by including an appropriate Chern-Simons term(axion coupling term with Dirac index density) in the bulk for odd(even) dimensions if the effective 4D gauge anomalies from the bulk and fixed points fermions cancel.

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